LONG WAVE PROPAGATION IN A TRIANGULAR CHANNEL

by

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THESIS

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OCEANOGRAPHY

from the

NAVAL POSTGRADUATE SCHOOL
October 1969

ABSTRACT

The purpose of this paper is to examine long wave propagation in a shallow channel of triangular cross section. Solutions for small scale $(f = 0)$, large scale $(f \neq 0)$, symmetric and asymmetric channels are obtained. Results are shown to be consistent with earlier work for the fundamental mode $(M = 0)$, and with edgewave solutions over a gently sloping bottom for the higher modes. A second class of waves (quasigeostrophic waves) is also obtained when the Coriolis effect is included. The results are compared with those for a rectangular channel.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

h_0	maximum depth of channel
g	gravitational acceleration
(u, v)	velocity components
(U, V)	x-variation of velocity components
ζ	surface height
η	x-variation of surface height
s	slope
n	slope factor $(n \geq 1)$
σ	frequency
f	Coriolis parameter
m	wave number
ξ	transformed lateral distance
W_0	non-dimensional wave number
α	non-dimensional frequency (squared)
F_a	Laguerre polynomial with argument a
(A, B)	arbitrary constants
c	wave celerity
ω	non-dimensional frequency
M	wave mode
K	channel scale parameter

ACKNOWLEDGEMENT

This problem was suggested by Professor Theodore Green. His advice and assistance were invaluable and made the results contained in this study possible.

I. INTRODUCTION

Investigations of long wave propagation in channels with cross sections which don't vary along the channel began with Kelland [1839], who found solutions for progressive waves in a uniform channel of triangular cross section with sides inclined at 45° to the horizontal. MacDonald [1894] examined wave propagation in a triangular channel with sides inclined at 30° to the horizontal. Most investigators have limited their work to small scale channels (i.e. the Coriolis effect was neglected as in the above cases) in which velocities normal to the axis of the channel were zero. An exception is Dronkers [1964], who studied waves in a rotating rectangular channel, allowing cross-channel motion.

Many rivers, estuaries, and shallow seas can be approximately represented by a shallow triangular channel (at least this is more realistic than a rectangular channel). The shallow-water solutions for long waves progressing along such a channel are closely related to the well known solutions for edgewaves [Ursell, 1952; and Reid, 1958]. Such a solution is carried out in this paper and the results compared to the work of Dronkers. Most of the results are obtained numerically. When possible, they are compared to asymptotic work.

II. ANALYSIS

Long waves are studied, propagating along a channel with a uniform triangular cross section. The coordinate system is shown in Figure 1.

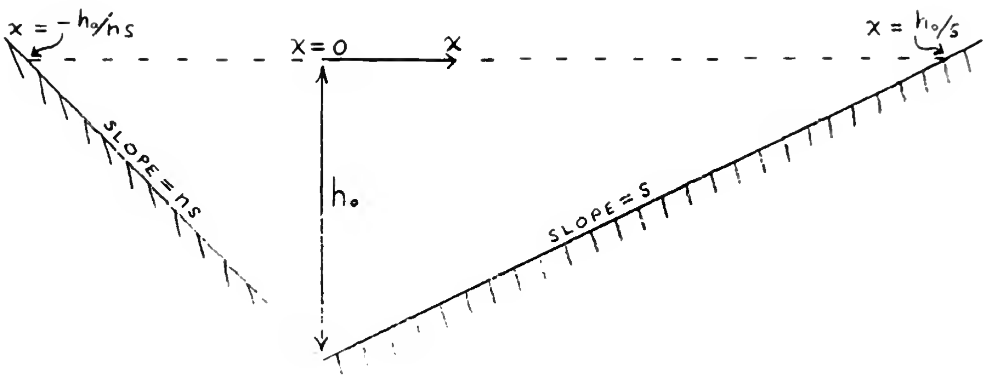

Figure 1

The maximum depth of the channel is h_0. The slopes of the channel sides are s for $x > 0$ and ns $(n \geq 1)$ for $x < 0$ (under the shallow water approximation $s, ns << 1$). The depth then is

$$h(x) = h_0 - sx \qquad (x > 0) \tag{1}$$

$$h(x) = h_0 + nsx \qquad (x < 0). \tag{2}$$

The equations to be satisfied are the shallow-water equations of motion

$$\frac{\partial u}{\partial t} - fv + g\frac{\partial \zeta}{\partial x} = 0 \tag{3}$$

$$\frac{\partial v}{\partial t} + fu + g\frac{\partial \zeta}{\partial y} = 0, \tag{4}$$

and the continuity equation

$$\frac{\partial}{\partial x}(hu) + h\frac{\partial v}{\partial y} + \frac{\partial \zeta}{\partial x} = 0 \tag{5}$$

where $u = x$ - component of velocity

$v = y$ - component of velocity

ζ = surface height

and y is along the channel. A wave motion in the y direction may be prescribed by the substitutions

$$
\begin{aligned}
\zeta &= \eta(x)\, e^{i(my+\sigma t)} \\
u &= U(x)\, e^{i(my+\sigma t)} \\
v &= V(x)\, e^{i(my+\sigma t)}
\end{aligned}
\tag{6}
$$

where m is the wave number, and σ the frequency. Then for $\sigma^2 \neq f^2$,

$$
\begin{aligned}
U &= \frac{ig}{\sigma^2 - f^2}\left(mf\eta + \sigma \frac{d\eta}{dx}\right) \\
V &= \frac{-g}{\sigma^2 - f^2}\left(m\sigma\eta + f \frac{d\eta}{dx}\right)
\end{aligned}
\tag{7}
$$

from equations (3) and (4). Substitution into the continuity equation yields

$$
h(x)\frac{d^2\eta}{dx^2} + \frac{dh}{dx}\frac{d\eta}{dx} + \left(\frac{mf}{\sigma}\frac{dh}{dx} + \frac{\sigma^2 - f^2}{g} - m^2 h\right)\eta = 0 \tag{8}
$$

for all x and any $h(x)$.

Equation (8) will now be dealt with in the regions $x > 0$ and $x < 0$ separately, and the results will be patched to insure wave-height and velocity continuity at $x = 0$.

A. EQUATION FOR $x > 0$

Let $\xi = h_0/s - x$. Then $d/dx = -d/d\xi$, $d^2\eta/dx^2 = d^2\eta/d\xi^2$, and $h(x) = h_0 - sx = s\xi$. The solution region is shown in Figure 2.

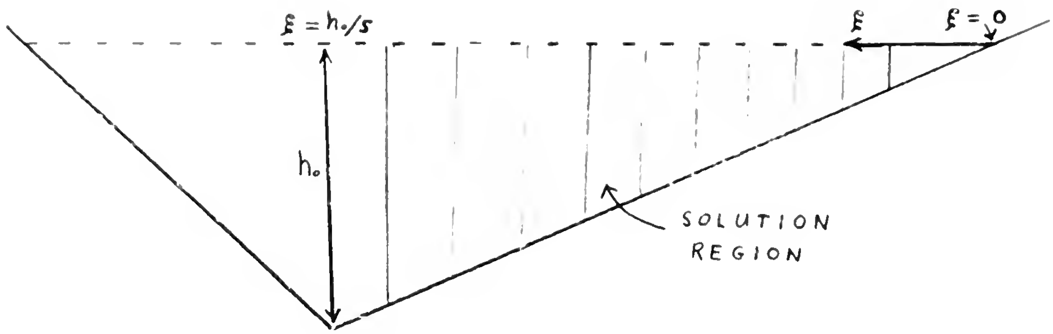

Figure 2

Then (8) becomes

$$\xi \frac{d^2\eta}{d\xi^2} + \frac{d\eta}{d\xi} + (P - m^2\xi)\eta = 0 \tag{9}$$

where
$$P = \frac{\sigma^2 - f^2}{g s} - \frac{m f}{\sigma} . \tag{10}$$

B. EQUATION FOR $x < 0$

Let $\xi' = x + h_o/ns$. Then $h(x) = h_o + nsx = ns\xi'$ and the corresponding solution region is shown in Figure 3.

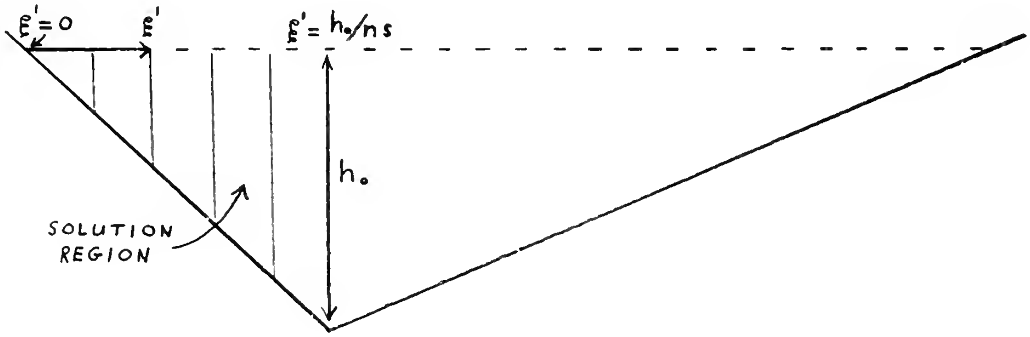

Figure 3

Then (8) becomes

$$\xi' \frac{d^2\eta}{d\xi'^2} + \frac{d\eta}{d\xi'} + (P' - m^2\xi')\eta = 0 \tag{11}$$

where
$$P' = \frac{\sigma^2 - f^2}{g n s} + \frac{m f}{\sigma} . \tag{12}$$

C. SOLUTION

Using the substitutions

$$\eta(\xi) = e^{-m\xi} G \qquad (x > 0)$$
$$\eta(\xi') = e^{-m\xi'} G' \qquad (x > 0) \tag{13}$$

equation (9) and (11) are put into hypergeometric form (Slater [1960]):

$$W \frac{d^2G}{dW^2} + (1-W)\frac{dG}{dW} + \left(\frac{p-m}{2m}\right) G = 0 \qquad (x > 0) \tag{14}$$

$$W' \frac{d^2G'}{dW'^2} + (1-W')\frac{dG'}{dW'} + \left(\frac{p'-m}{2m}\right) G' = 0 \qquad (x < 0) \tag{15}$$

where $W = 2\xi m$, and $W' = 2\xi' m$.

These equations have solutions of the form

$$G = A F_a(W) + C H(W) \qquad (x > 0)$$
$$G' = B F_{a'}(W') + D H'(W') \qquad (x < 0) \tag{16}$$

where

$$a = \frac{p-m}{2m}$$
$$a' = \frac{p'-m}{2m} \tag{17}$$

But H, H' (U in Slater's notation) are unbounded as ξ and ξ' become small and must be discarded. Then the solutions of (14) and (15) become

$$\eta(\xi) = A e^{-m\xi} F_a(W) \qquad (x > 0) \tag{18}$$

$$\eta(\xi') = B e^{-m\xi'} F_{a'}(W') \qquad (x < 0) \tag{19}$$

where

$$F_a(W) = 1 + aW + \frac{a(a+1)}{(2!)^2}W^2 + \frac{a(a+1)(a+2)}{(3!)^2}W^3 \cdots .$$

$$F_{a'}(W') = 1 + a'W' + \frac{a'(a'+1)}{(2!)^2}W'^2 + \frac{a'(a'+1)(a'+2)}{(3!)^2}W'^3 \cdots .$$

The quantities A and B are constants; we can set $A = 1$ as the results will be independent of wave amplitude in this linear theory.

At $x = 0$ the patching conditions are:

$$\eta(\xi) = \eta(\xi') \quad \text{at } x = 0 \quad (W = W_0,\ W' = W_0') \tag{20}$$

(i.e., the wave-height is continuous), and

$$U = U' \quad \text{at } x = 0 \quad (W = W_0,\ W' = W_0') \tag{21}$$

(i.e., the cross-channel velocity component is continuous). Note that $W_0 = 2mh_0/s$ and $W_0' = 2mh_0/ns = W_0/n$ are convenient nondimensional wave numbers.

Using (20), (21) becomes

$$\left(\frac{d\eta}{dx}\right)_{x=0^-} = \left(\frac{d\eta}{dx}\right)_{x=0^+} \tag{22}$$

or

$$\left(\frac{d\eta}{d\xi}\right)_{\xi=h_0/s} + \left(\frac{d\eta}{d\xi'}\right)_{\xi'=h_0/ns} = 0. \tag{23}$$

Then, using (7), (20), and (23), V is also continuous at $x = 0$. For $F_a(W_0) = 0$, (20) gives: 1) $B = 0$ which means (21) cannot be satisfied or 2) $F_{a'}(W_0') = 0$ which corresponds to a node at $x = 0$. For $F_a(W_0) \neq 0$, equations (20) and (23) give, eliminating B,

$$F_a(W_0)\,F_a'\left(\frac{W_0}{n}\right) - F_a(W_0)\,\frac{dF_a'}{dW'}\left(\frac{W_0}{n}\right) - F_a'\left(\frac{W_0}{n}\right)\frac{dF_a}{dW}(W_0) = 0 \,. \qquad (24)$$

Note that the solutions $F_a(W_0) = F_a'(W_0') = 0$ are contained in (24). Also, when the channel is symmetric $(n = 1.0)$, (24) becomes

$$F_a(W_0)\left[F_a(W_0) - 2\,\frac{dF_a}{dW}(W_0)\right] = 0 \,. \qquad (25)$$

III. SMALL-SCALE CHANNELS

For relatively short waves in narrow channels, where the Coriolis effect may be neglected, (10), (12), and (17) become

$$p = \frac{\sigma^2}{g s} = n p' \tag{26}$$

and

$$\frac{\sigma^2}{m g s} = 1 - 2a . \tag{27}$$

Equation (27) can be written

$$\alpha \equiv \frac{\sigma^2 h_0}{g s^2} = (1 - 2a)\frac{W_0}{2} \tag{28}$$

where $\alpha^{1/2}$ is a convenient nondimensional frequency. Similarly,

$$\frac{\sigma^2}{m g n s} = 1 - 2a' \tag{29}$$

and

$$\frac{\alpha}{n} = \frac{\sigma^2 h_0}{g n s^2} = (1 - 2a')\frac{W_0}{2} . \tag{30}$$

Equating like terms in (28) and (30) yields the following relationship between a and a':

$$(1 - 2a) = n(1 - 2a') . \tag{31}$$

Then equation (24) relates nondimensional wave numbers W_0 and frequencies α.

A. SYMMETRICAL CHANNEL $(n = 1.0)$

1. Asymptotic Results

For channels with sides of equal slope (31) gives $a = a'$. If $W_0 << 1$, and a is bounded, an asymptotic result can be obtained. Dropping terms of $O(W_0^2)$ and higher in (25) gives

$$(1-2a) \approx W_0 \left[2a^2 + a(a-1) \right] \qquad (W_0 << 1) \tag{32}$$

so that $a \approx 1/2$. Thus, using (28), (32) gives

$$W_0 = \frac{2mh_0}{s} \approx \frac{4\sigma^2}{mgs} \qquad (W_0 << 1) \tag{33}$$

or

$$8\alpha \approx W_0^2 \qquad (W_0 << 1) . \tag{34}$$

Since wave celerity $c = \sigma/m$, this can be written

$$c^2 = 1/2 \, g h_0 \tag{35}$$

which is identical to Kelland's result for a symmetric triangular channel. Since the average depth is $h_0/2$, then this result also corresponds to the classical result for long waves over a flat bottom: $c^2 = gh$.

Other asymptotic results for $W_0 << 1$ and a unbounded can be obtained by iteration. An example is given below.

Let W_N be the root obtained by keeping terms through W^N in $F_a(W) = 0$. Then:

$$-aW_1 = 1$$

$$-aW_2 = 1 + \frac{a(a+1)}{(2!)^2} W_1^2 = 1 + \frac{1}{(2!)^2} + O\left(\frac{1}{a}\right)$$

$$-aW_3 = 1 + \frac{a(a+1)}{(2!)^2} W_2^2 + \frac{a(a+1)(a+2)}{(3!)^2} W_2^3$$

$$= 1 + \frac{1}{(2!)^2}\left[1 + \frac{1}{(2!)^2}\right]^2 - \frac{1}{(3!)^2}\left[1 + \frac{1}{(2!)^2}\right]^3 + O\left(\frac{1}{a}\right)$$

$$-aW_4 = \text{etc.}$$

This converges rapidly to

$$-aW_o = 1.42 + O\left(\frac{1}{a}\right)$$

and from (28),

$$\alpha \approx 1.42 . \qquad (36)$$

Similar techniques could give the other asymptotes suggested by the numerical results presented below.

For $W_o >> 1$ (short waves), it is expected on a physical basis that the waves in the two parts of the channel become independent of one another, and that Ursell's edgewave result

$$\sigma^2 = (2M + 1) \qquad (M = 0, 1, 2, \ldots) \qquad (37)$$

would hold. Although this has not been confirmed here analytically, it is borne out by the numerical results.

2. Numerical Results

A Fortran IV computer program to solve (24) for small scale channels $(f = 0)$ is given in Appendix A. The results for the symmetric channel $(n = 1.0)$ are shown in Figures 4 and 5. In Figure 5, the dispersion relation for the fundamental mode $(M = 0)$ and the next five modes are plotted.

For $W_o << 1$ (Figure 4) the calculations agree well with the asymptotic results (34) and (36).

For $W_o >> 1$, note that

$$\alpha \approx (2M + 1)\frac{W_o}{2} \qquad (M = 0, 1, 2, \ldots)$$

or

$$\alpha \approx (2M + 1)\, g\, m\, s \qquad (M = 0, 1, 2, \ldots)$$

which is the same as (37).

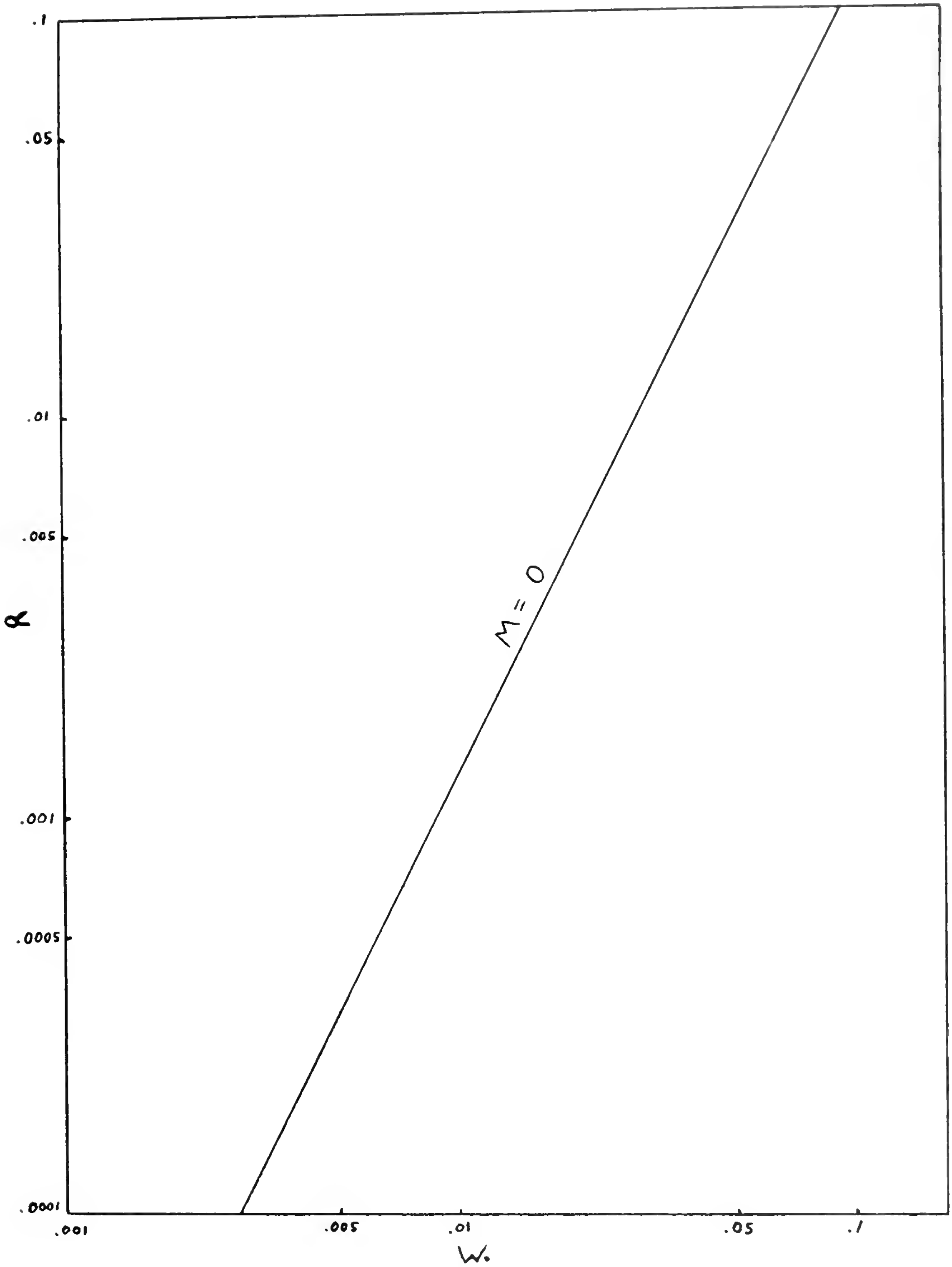

Figure 4

Dispersion Relation: Symmetric Channel ($W_0 << 1$)
($f=0, n=1.0$)

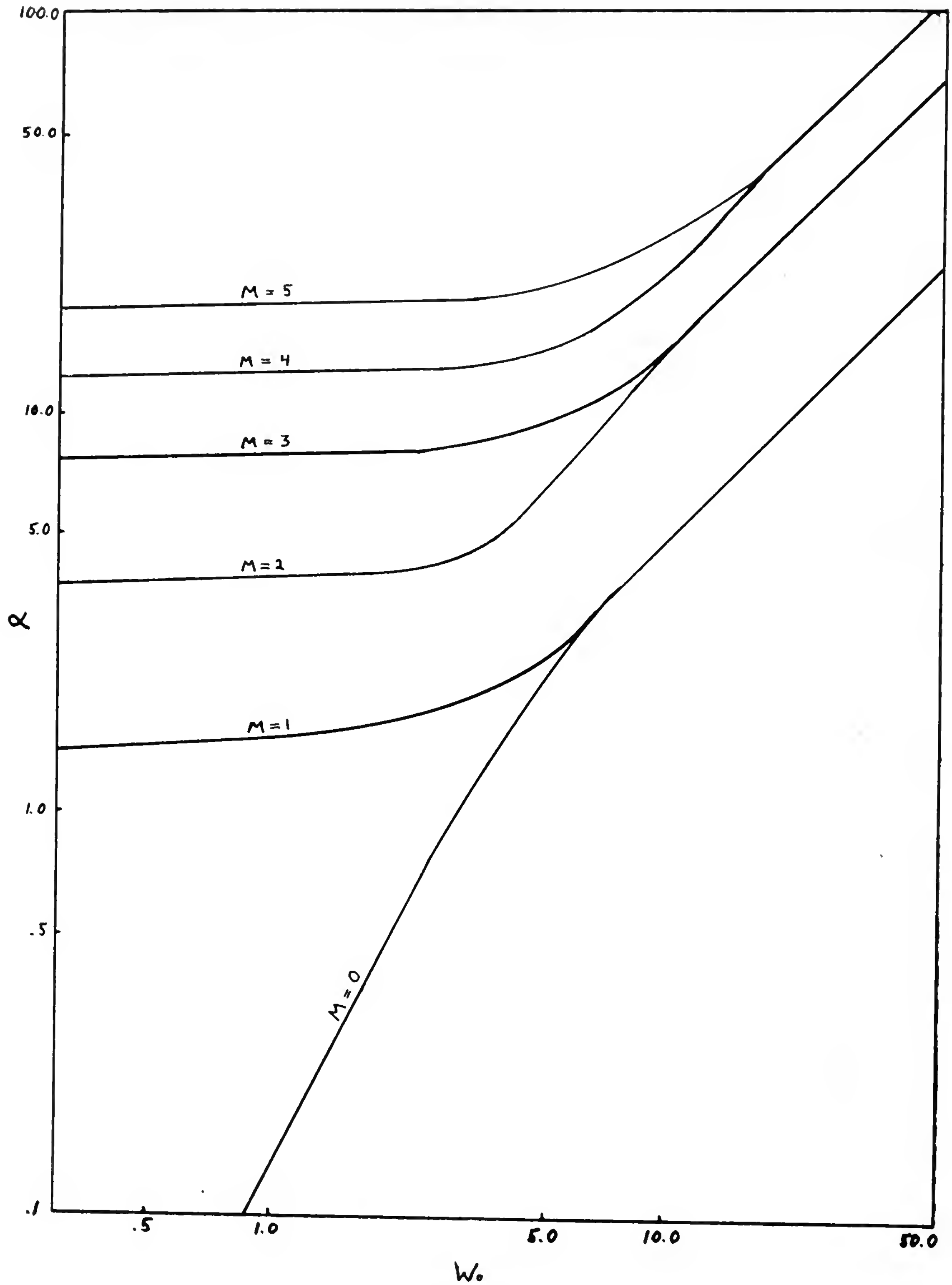

Figure 5

Dispersion Relation: Symmetric Channel (f=0,n=1.0)

3. Wave Profiles

Figure 6 is a sketch of the wave profiles for modes 0 through 5 showing the nodal points for the symmetrical channel. Note that the nodes for all modes are symmetric about the center of the channel. The profile for the fundamental mode $(M = 0)$ has no nodes along the channel. Wave mode $(M = 1)$ is the first asymmetric profile, corresponding to the solution $\eta(\xi) = -\eta(\xi'), [F_a(W_0) = 0]$. Wave mode $(M = 2)$ has a symmetric profile with two nodal points. The higher modes have symmetric profiles for even mode numbers and asymmetric profiles for odd (M), with the number of nodal points equal to (M). The nodal points (except $x = 0$) move toward the channel walls with increasing W_0. This corresponds to the edge-wave condition for short waves (large m).

B. ASYMMETRICAL CHANNEL $(n \neq 1.0)$

1. Asymptotic Results

For $W_0 << 1$ and a bounded, (24) can be written

$$1 - (a + a') = W_0\left[aa'\left(\frac{n+1}{n}\right) + \frac{a(a-1)}{2} + \frac{a'(a'-1)}{2n}\right] + O(W_0^2) . \tag{38}$$

Since $W_0 << 1$, then $a + a' \approx 1$. Using (31), this gives

$$a + \frac{1}{2n}\left(2a + n - 1\right) \approx 1$$

so that

$$a \approx 1/2 ; \quad a' \approx 1/2 .$$

Let $a = 1/2 - \epsilon$ and $a' = 1/2 - \epsilon'$. Then (38) gives

$$W_0 \approx \frac{8n}{n+1}\left(\epsilon + \epsilon'\right) \tag{39}$$

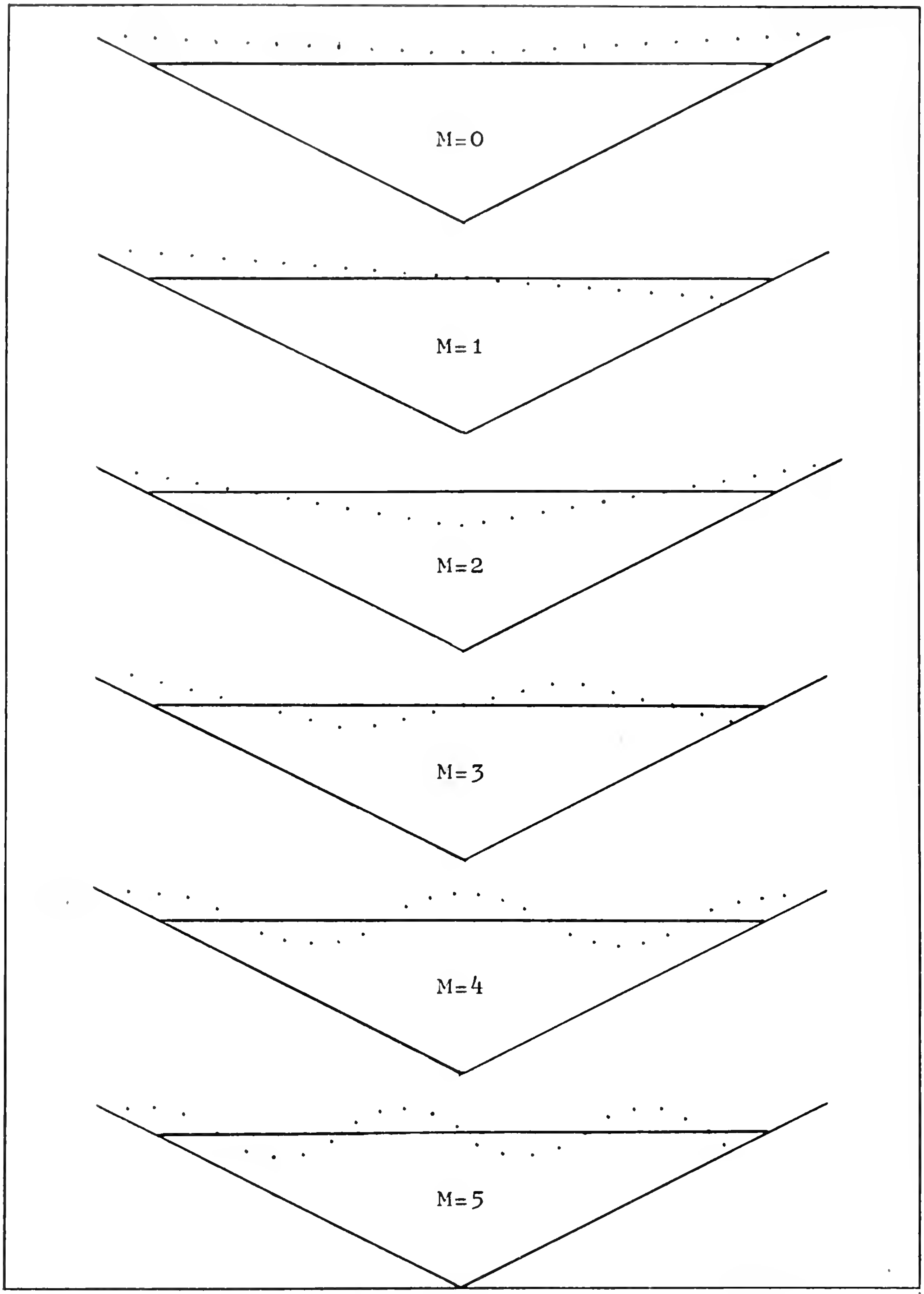

Figure 6

Wave Profiles: Symmetric Channel (n=1.0) which with (28) and (30) gives

$$W_0^2 = 8\alpha .$$

This is the same as (34), and is independent of n.

Other results for $W_0 << 1$ similar to (36) are expected for asymmetric channels but are not examined here.

2. Numerical Results

The results for $n = 2.0$ and $n = 5.0$ are shown in Figures 7 and 8. In each case, the asymptotic result (34) is confirmed. In fact, the entire dispersion relation for the fundamental mode $(M = 0)$ appears to be independent of n. The higher modes have all been shifted to higher frequencies in comparison with the symmetric case.

3. Wave Profiles

Profiles for $n = 2.0$ and $n = 5.0$ are shown in Figures 9 and 10. Again the fundamental mode $(M=0)$ is unchanged, but the higher modes have been altered significantly from the symmetric case.

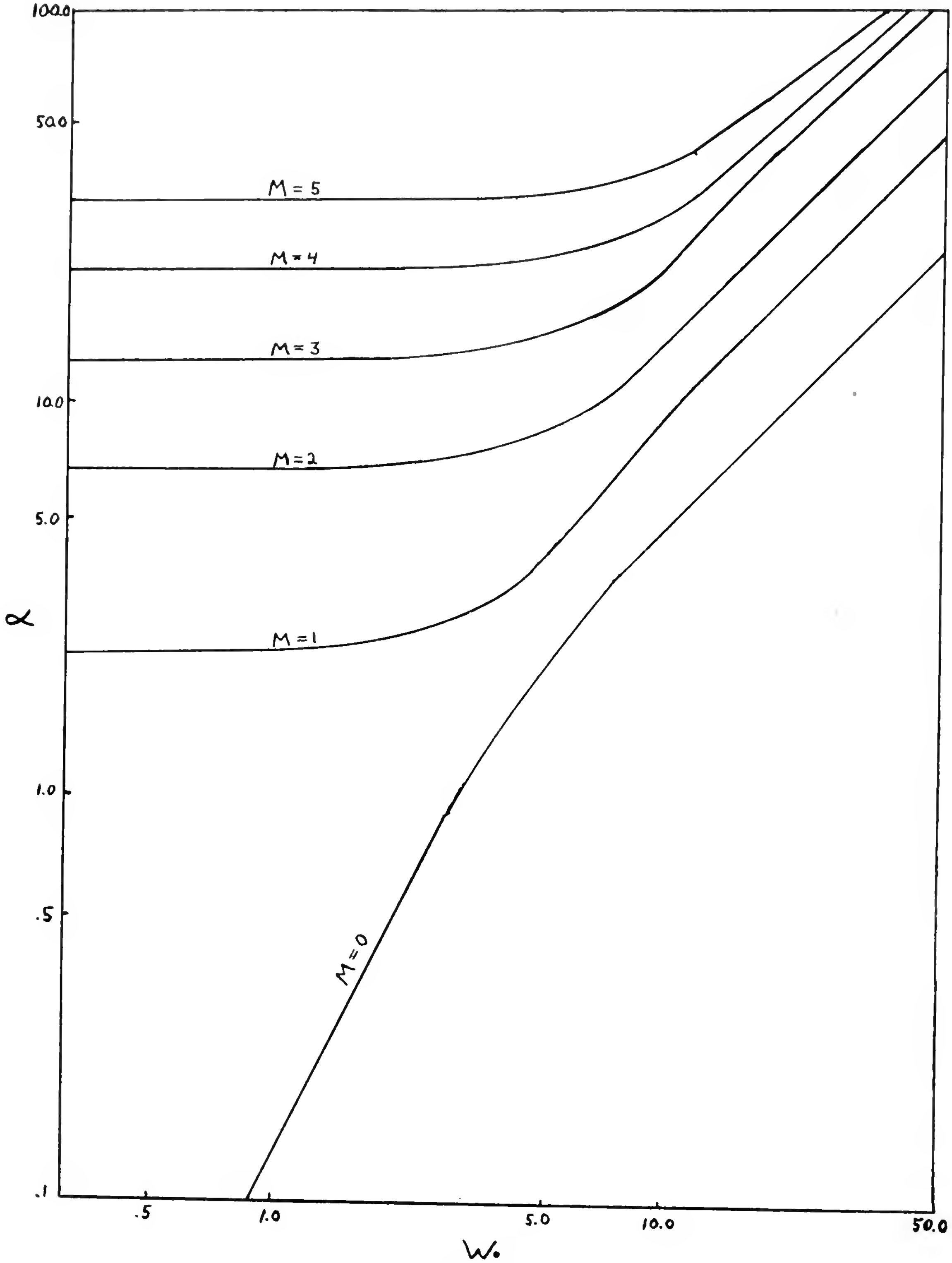

Figure 7

Dispersion Relation: Asymmetrical Channel (f=0,n=2.0)

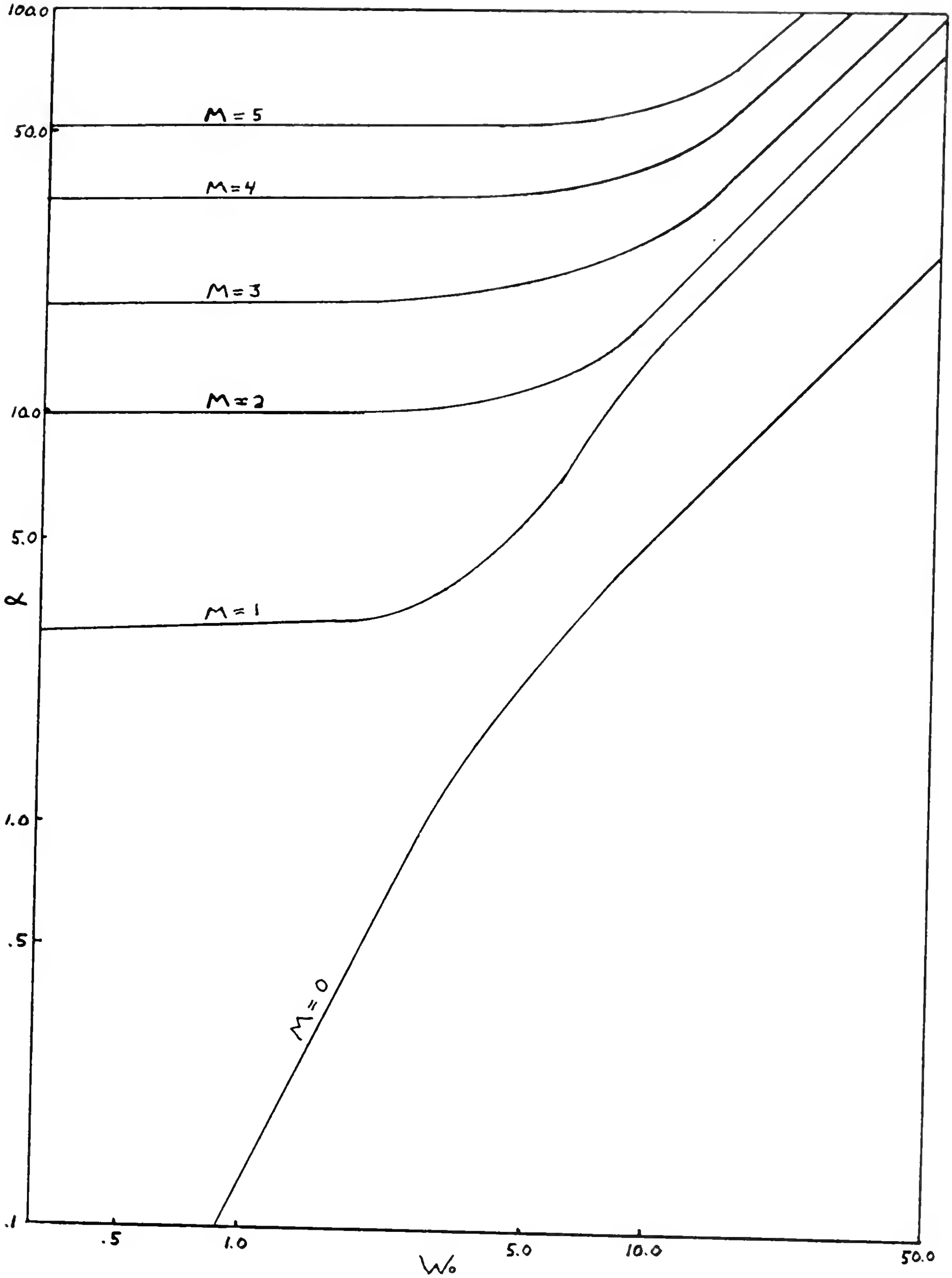

Figure 8

Dispersion Relation: Asymmetrical Channel (f=0,n=5.0)

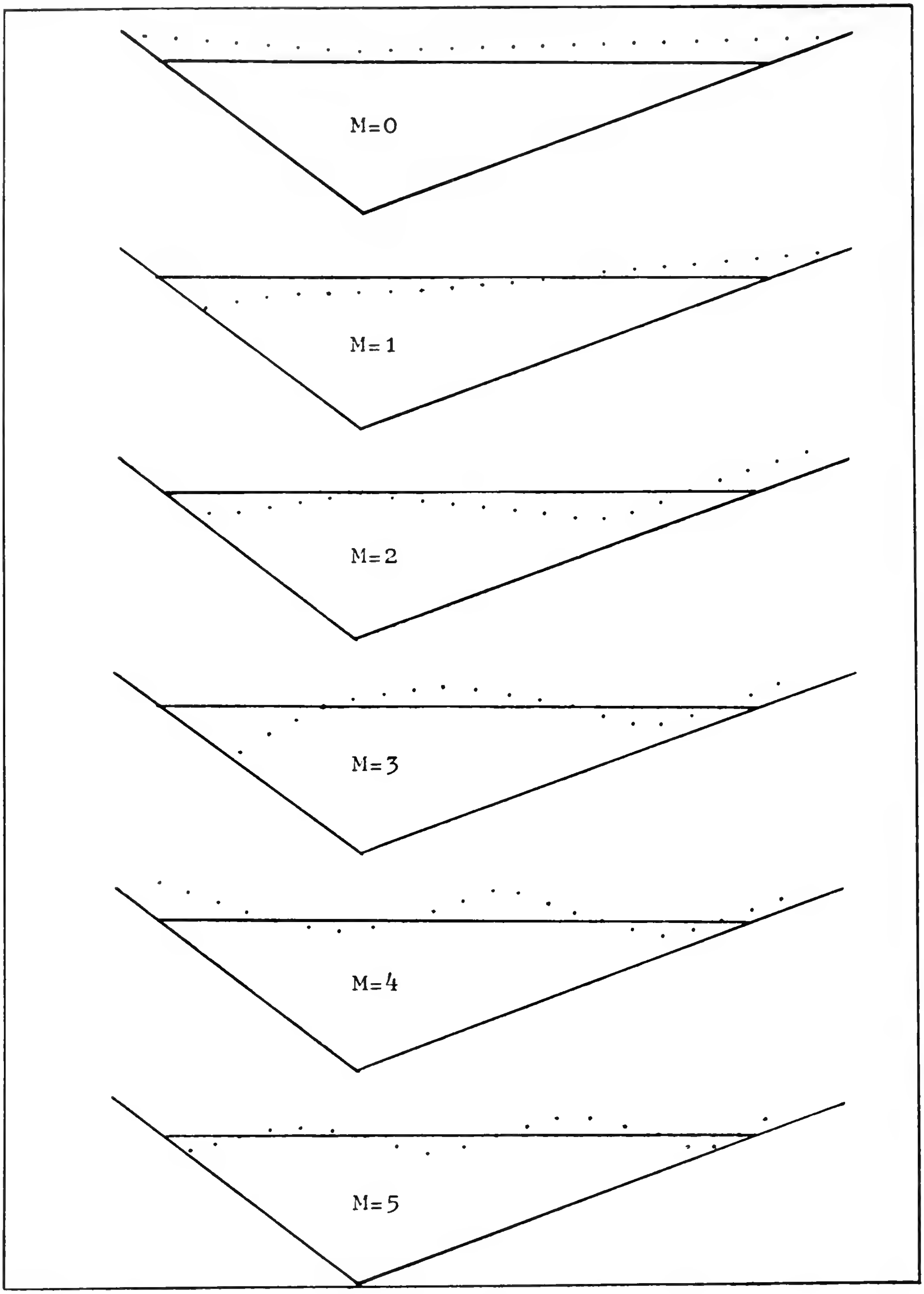

Figure 9

Wave Profiles: Asymmetric Channel (n=2.0)

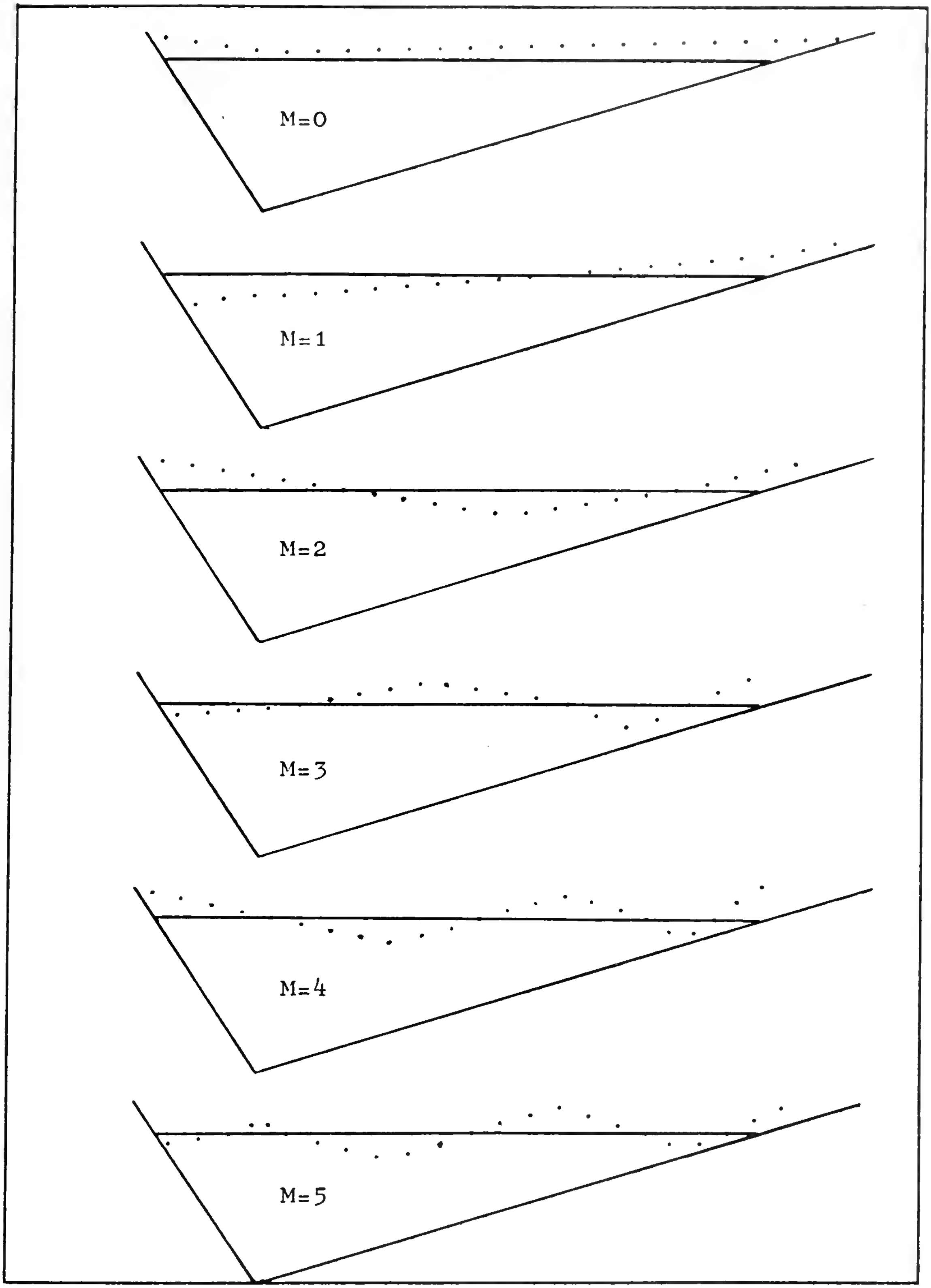

Figure 10

Wave Profiles: Asymmetric Channel (n=5.0)

IV. LARGE-SCALE CHANNELS

For large channels where the Coriolis effect must be considered, (10), (12), and (17) give

$$\mu \equiv 1 - 2a = \frac{f^2}{gms}(\omega^2 - 1) - \frac{1}{\omega}$$

and (40)

$$\mu' \equiv 1 - 2a' = \frac{f^2}{gmns}(\omega^2 - 1) + \frac{1}{\omega}$$

where

$$\omega = \sigma/f .$$

Equating like terms in (40),

$$\omega = \frac{n+1}{n\mu' - \mu} \tag{41}$$

and

$$W_o = \frac{K}{n}\left(\frac{n+1}{\mu+\mu'}\right)\left[\left(\frac{n+1}{n\mu'-\mu}\right)^2 - 1\right] \tag{42}$$

where

$$K = \frac{2f^2 h_o}{g s^2} .$$

Now W_o must satisfy both (24) and (42).

A. SYMMETRICAL CHANNEL $(n = 1.0)$

1. Asymptotic Results

From (41), a and a' are related by $a = a' + 1/\omega$. But for $W_o << 1$ and ω bounded, (42) implies that $|a + a'|$ is large, so that $a \approx a'$. Then (40) gives

$$-2a = \frac{f^2}{gsm}(\omega^2 - 1) - \frac{1}{\omega}. \tag{43}$$

This can be written

$$-2a = \frac{K}{W_o}(\omega^2 - 1) - \frac{1}{\omega}. \tag{44}$$

From (36) and (44),

$$\frac{2.84}{W_0} = \frac{K}{W_0}(\omega^2 - 1) - \frac{1}{\omega} \qquad (W_0 << 1)$$

Dropping the small term gives

$$\omega^2 = \frac{2.84}{K} + 1 . \tag{45}$$

2. Numerical Results

A Fortran IV computer program to solve (24) and (42) is given in Appendix B. A value of $K = 0.1$ is chosen, which corresponds approximately to the dimensions of the Red Sea. The results for the symmetric channel are shown in Figure 11. Note that for $W_0 << 1$, the asymptotic result (45) is confirmed. For $\omega << 1$ a new class of waves appears. These are analogous to the quasigeostrophic edgewaves obtained by Reid. Figure 12 is an expanded plot of these low frequency waves.

For $\omega, W_0 >> 1$ (short high-frequency waves), the Coriolis effect should be negligible. Then, for a fixed W_0, the frequencies σ for $f = 0$ and $f \neq 0$ should be the same. Thus,

$$\frac{\alpha}{\omega^2} = \frac{\sigma^2 h_0}{g s^2} \Big/ \frac{\sigma^2}{f^2} = \frac{K}{2} \tag{46}$$

This relation is observed to hold for $\omega, W_0 >> 1$.

B. ASYMMETRICAL CHANNEL ($n \neq 1.0$)

1. Asymptotic Results

For $W_0 << 1$ and a unbounded, asymptotic results similar to (45) are expected, but are not derived here.

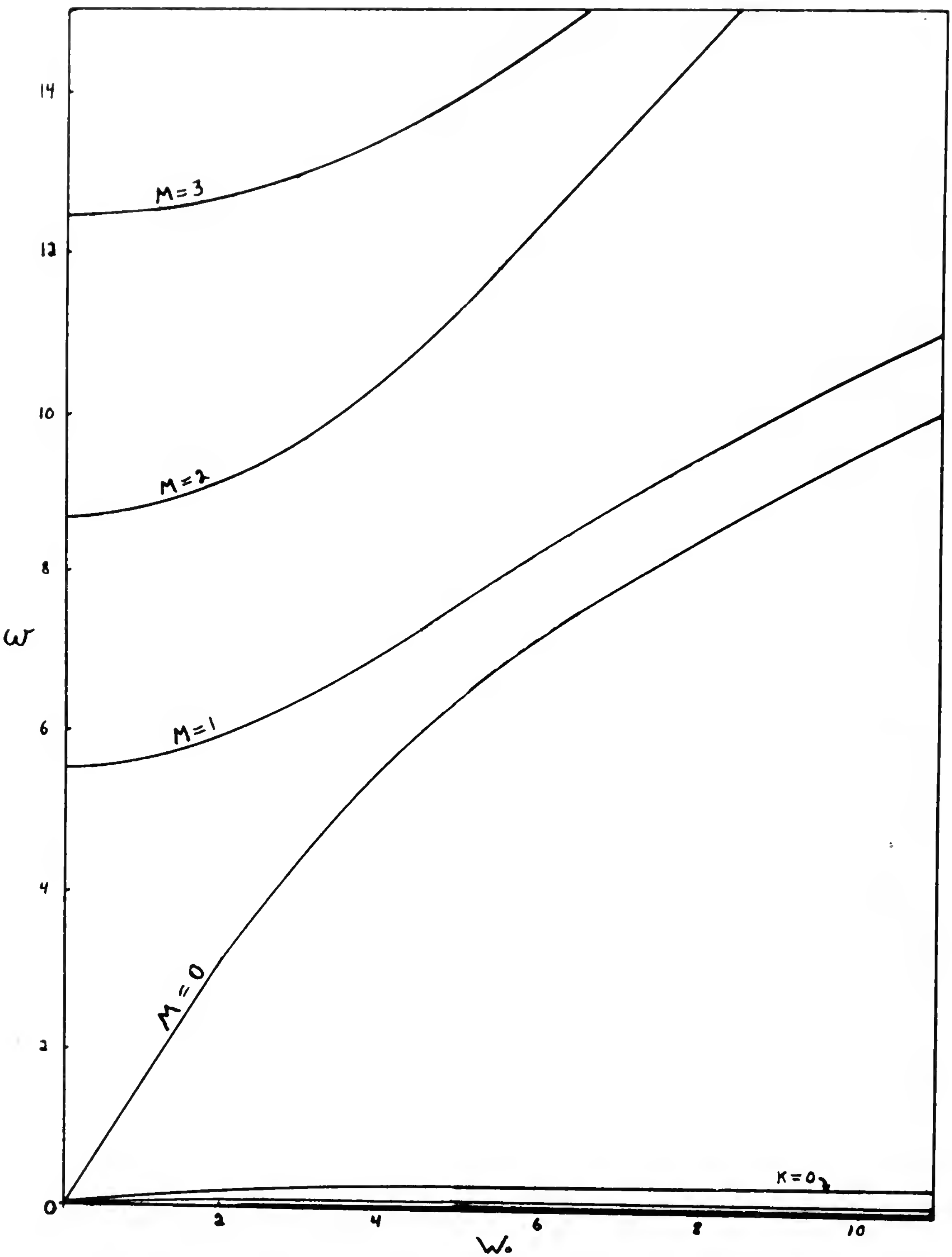

Figure 11

Dispersion Relation: Symmetric Channel (f=0,n=1.0)

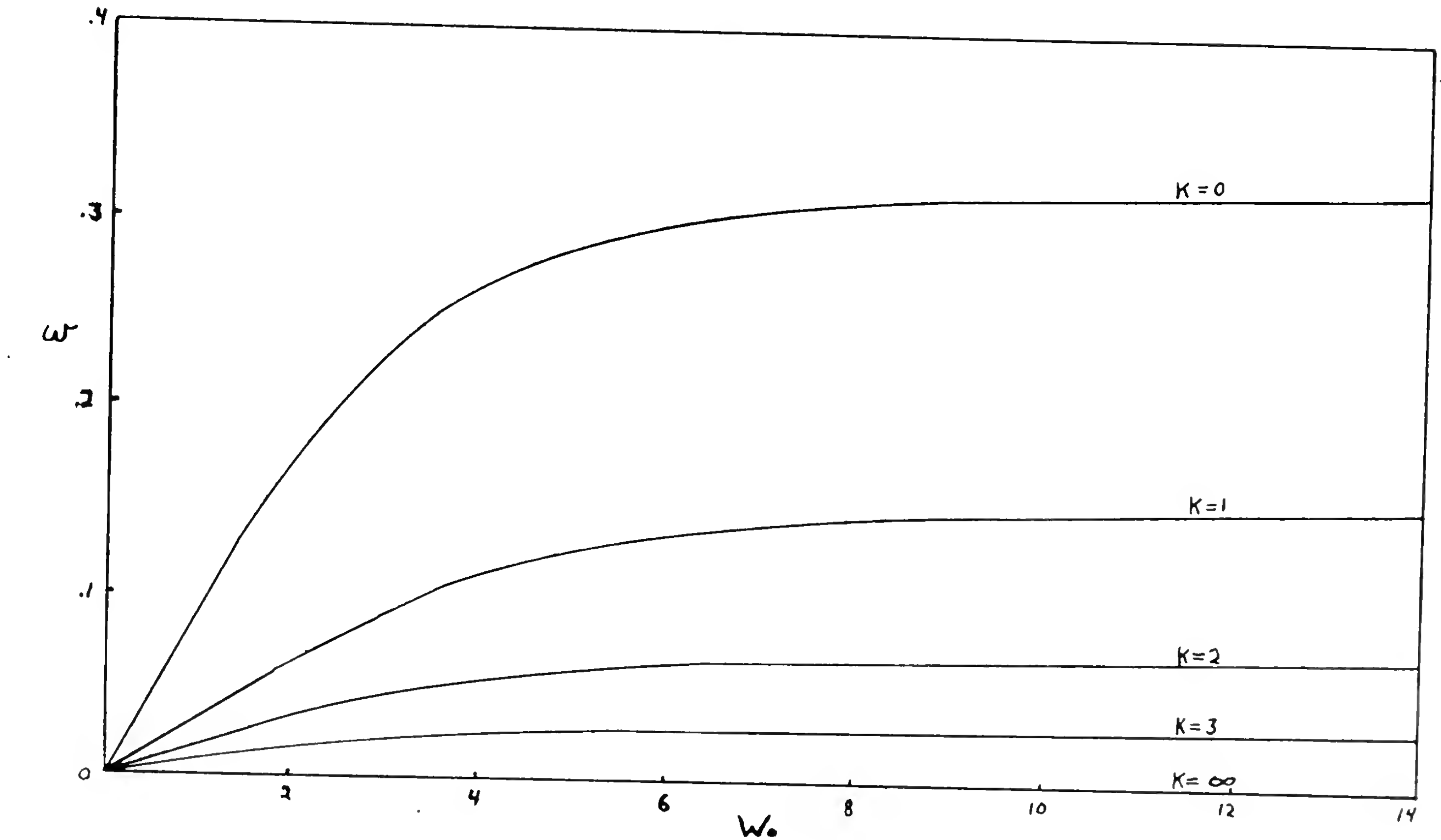

Figure 12

Dispersion Relation: Quasigeostrophic Waves (f=0,n=1.0)

2. <u>Numerical Results</u>

The results for $n = 2.0$ and $n = 5.0$ are shown in Figures 13 and 14. Note that all modes have been shifted to higher frequencies in comparison with the symmetric case. Also for $\omega\tau, W_0 >> 1$ the $f = 0$ and $f \neq 0$ results give good agreement via (46).

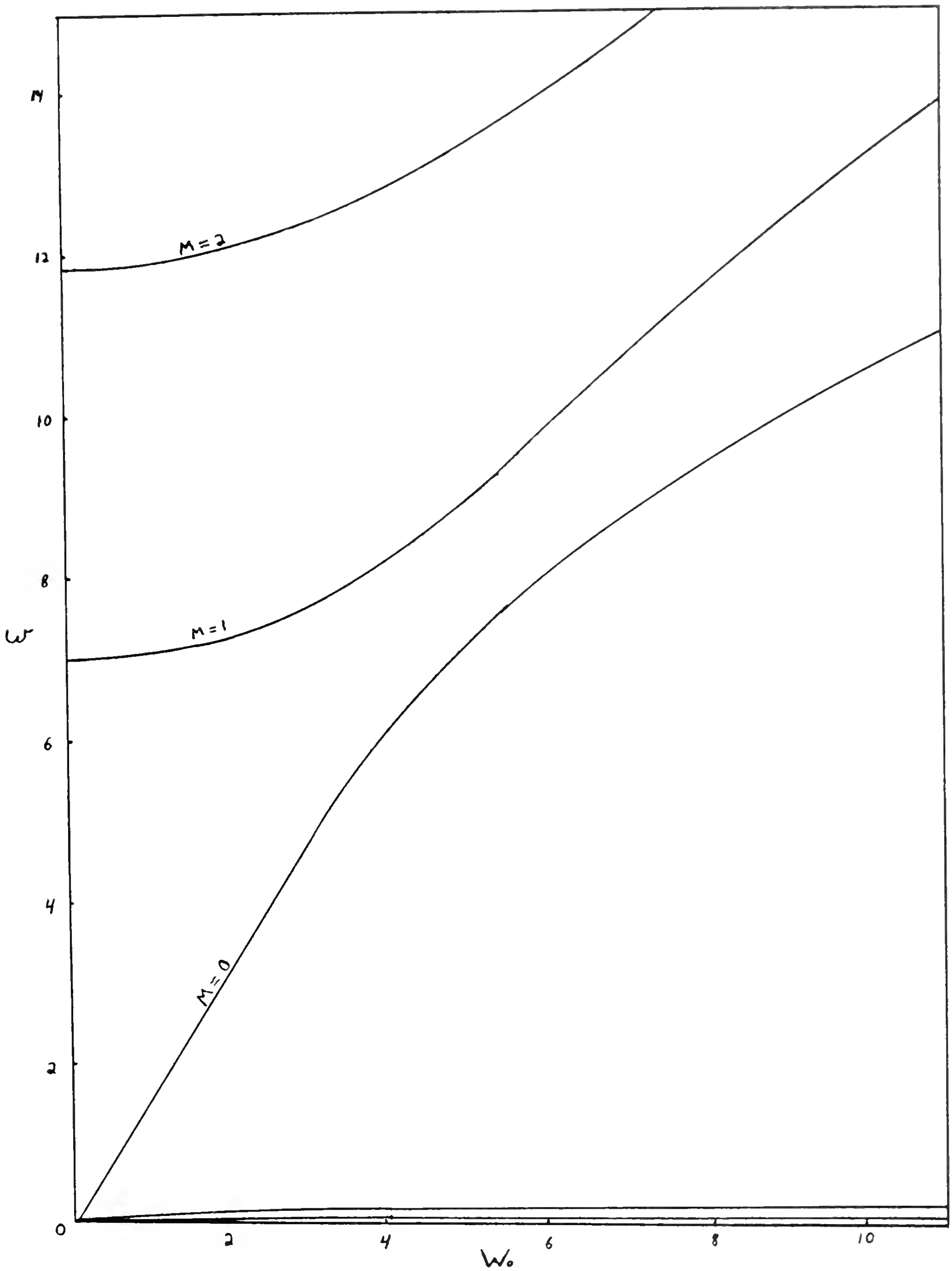

Figure 13

Dispersion Relation: Assymetric Channel (f≠0,n=2.0)

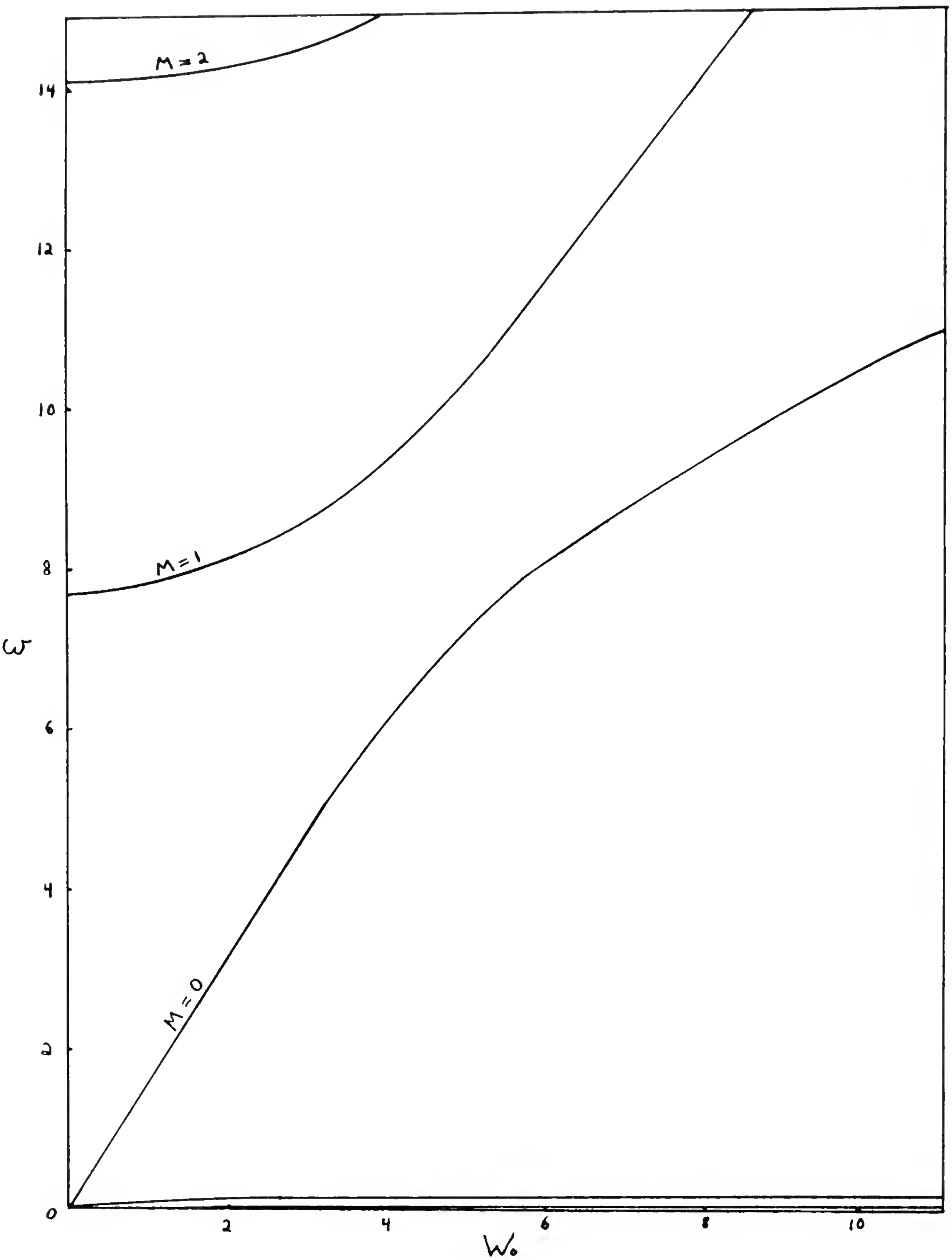

Figure 14

Dispersion Relation: Asymmetric Channel (f≈0,n=5.0)

V. COMPARISON OF RESULTS FOR A RECTANGULAR CHANNEL AND A SYMMETRIC TRIANGULAR CHANNEL

The dispersion relation due to Dronkers for a channel of uniform depth (h) and width (W) is (Appendix C)

$$m^2 = \frac{\sigma^2 - f^2}{gh} - \frac{M^2\pi^2}{W^2} \qquad (M = 0, 1, 2, \ldots). \tag{47}$$

It is reasonable to compare channels of equal average depth, (Figure 15).

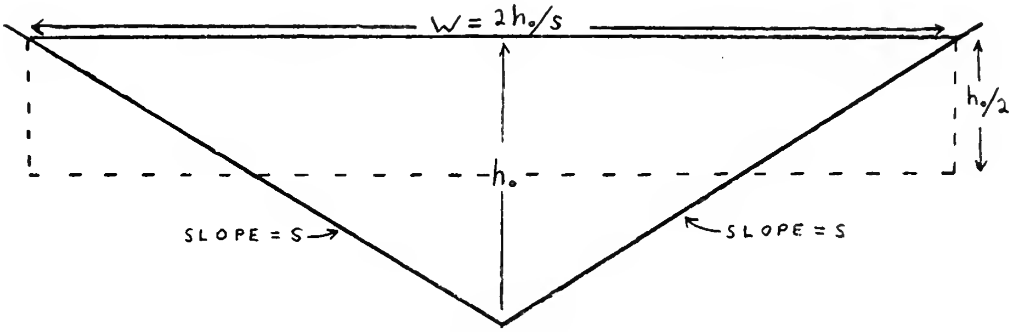

Figure 15

Then $h = h_0/2$ and $W = 2h_0/s$, and (47) becomes

$$W_0^2 = \frac{8h_0}{g s^2}(\sigma^2 - f^2) - M^2\pi^2. \tag{48}$$

A. SMALL-SCALE $(f = 0)$

For small-scale channels $(f = 0)$, (48) becomes

$$W_0^2 = 8\alpha - M^2\pi^2 \qquad (M = 0, 1, 2, \ldots). \tag{49}$$

This agrees with (34) for the fundamental mode $(M = 0)$. This dispersion relation (49) is compared with that for a symmetric triangular channel in Figure 16.

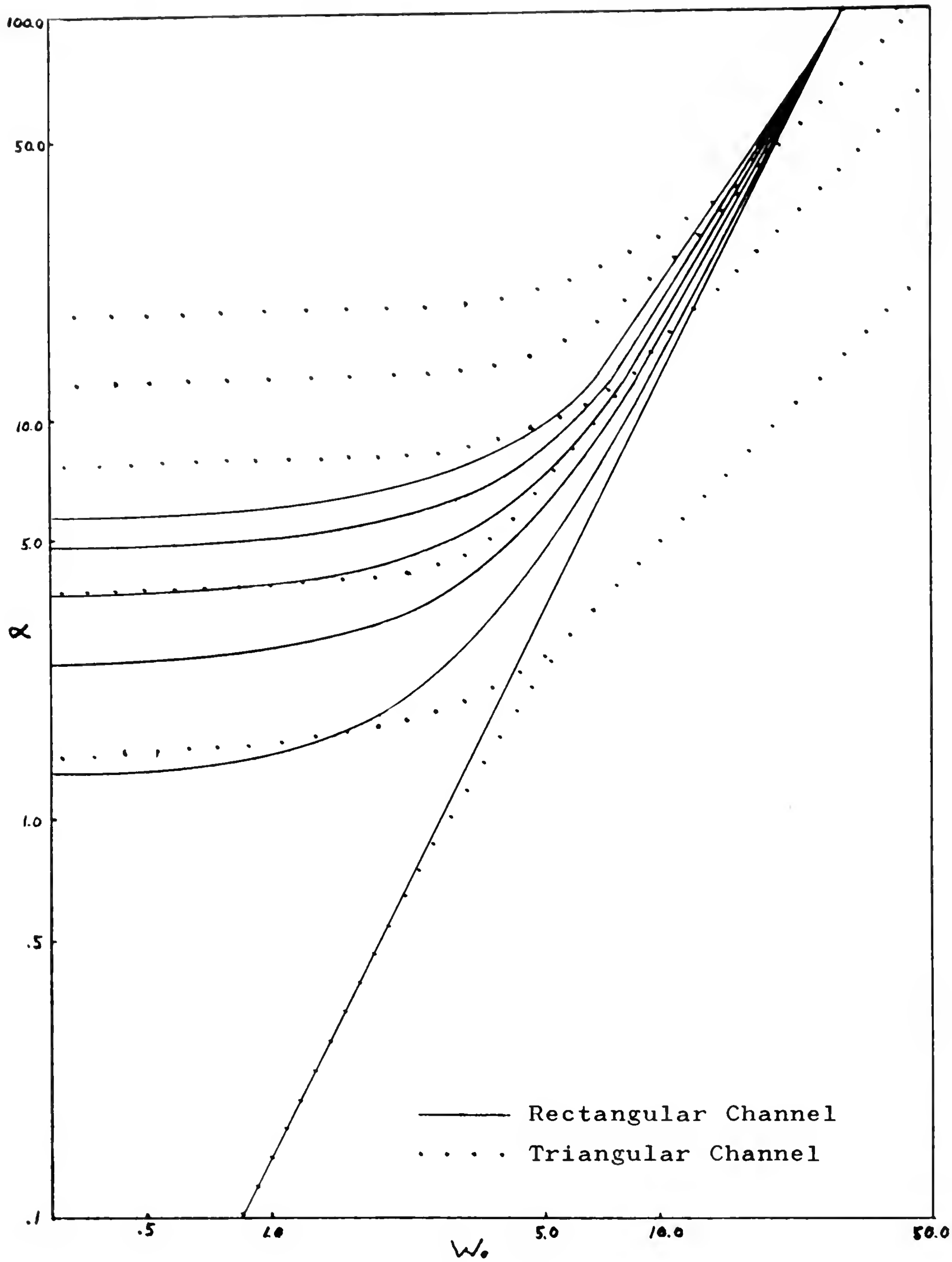

Figure 16

Dispersion Relations: Symmetric Triangular vs. Rectangular (f=0)

B. LARGE-SCALE $(f \neq 0)$

For large-scale channels $(f \neq 0)$, (48) becomes

$$W_0^2 = 4\chi(\omega^2 - 1) - M^2\pi^2 \qquad (M = 0, 1, 2, \ldots). \tag{50}$$

This dispersion relation (50) is compared with that of a symmetric triangular channel in Figure 17.

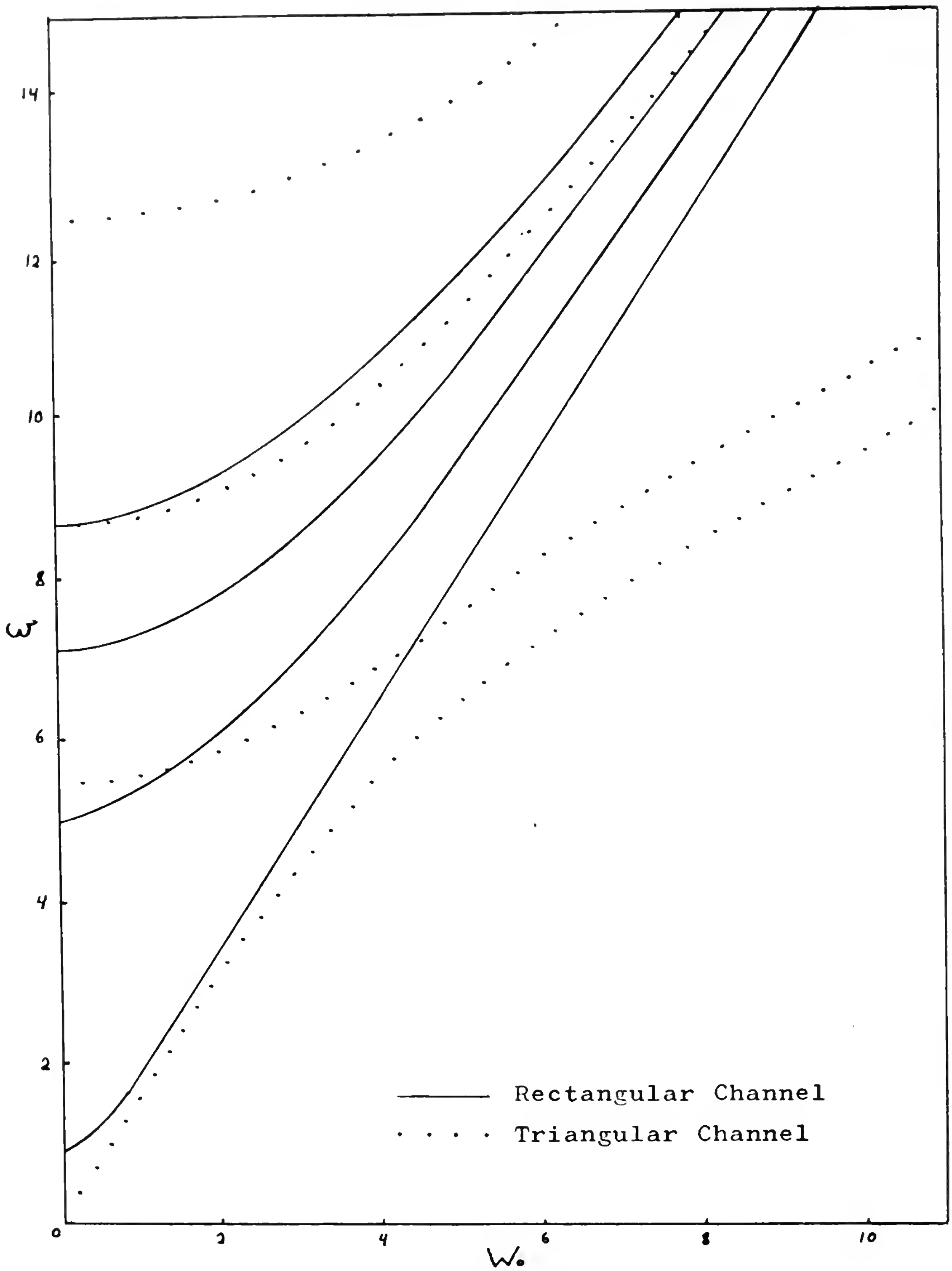

Figure 17

Dispersion Relations: Symmetric Triangular vs. Rectangular (f=0)

APPENDIX A

FORTRAN IV PROGRAM TO SOLVE PATCHING EQUATION FOR SMALL-SCALE CHANNELS

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION C(36),C(36),CP(36),DP(36),CCP(36),CDP(36),DCP(36),WORK
     *(36),B(36),ROOTR(36),ROOTI(36)
    1 READ(5,5)A
    5 FORMAT(F10.5)
      IF(A.EQ.-9.50)GOTO99
      S=5.0
      AP=(S+2.0*A-1.0)/(2.0*S)
      WRITE(6,10)A,AP,S
   10 FORMAT(35X,'A=',F10.5,5X,'AP=', F10.5,5X,'N=',F5.2,/)
      C(1)=1.0
      C(2)=A
      DO 2 N =3,12
      C(N)=C(N-1)*(A+(N-2))/((N-1)*(N-1))
    2 CONTINUE
      D(1)=A
      D(2)=A*(a+1)/2.0
      DO 3 N=3,12
      D(N)=D(N-1)*(A+(N-1))/((N-1)*N)
    3 CONTINUE
      CP(1)=1.0
      CP(2)=AP/S
      DO 4 N=3,12
      CP(N)=CP(N-1)*(AP+(N-2))/(N-1)*(N-1)*S)
    4 CONTINUE
      DP(1)=AP
      DP(2)=AP*(AP+1)/(2.0*S)
      DO 6 N=3,12
      DP(N)=DP(N-1)*(AP+(N-1))/((N-1)*N*S)
    6 CONTINUE
      DO 7 K=1,12
      CCP(K)=0.0
      DO 7 I=1,12
      DO 7 J=1,12
      IF(I+J.NE.K)GOTO7
      CCP(K-1)=CCP(K-1)+C(I)*CP(J)
    7 CONTINUE
      DO 8 K=1,12
      CDP(K)=0.0
      DO 8 I=1,12
      DO 8 J=1,12
      IF(I+J.NE.K)GOTO8
      CDP(K-1)=CDP(K-1)+C(I)*DP(J)
```

```
  8  CONTINUE
     DO 9 K=1,12
     DCP(K)=0.0
     DO 9 I=1,12
     DO 9 J=1,12
     IF(I+J.NE.K)GOTO9
     DCP(K-1)=DCP(K-1)+D(I)*CP(J)
  9  CONTINUE
     DO 11 N=1,11
     B(N)=CCP(N)=CDP(N)-DCP(N)
 11  CONTINUE
     CALL DPOLRT(B,WORK,10,ROOTR,ROOTI,IER)
     WRITE (6,25)(ROOTR(I),ROOTI(I),I=1,10)
 25  FORMAT(//,4X,2D25.16,/)
     WRITE(6,30)IER
 30  FORMAT(50X,I2)
     GOTO1
 99  STOP
     END
```

APPENDIX B

FORTRAN IV PROGRAM TO SOLVE PATCHING EQUATION FOR LARGE-SCALE CHANNELS

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION C(36),D(36),CP(36),DP(36),CCP(36),CDP(36),DCP(36),WORK
     *(36),B(36),ROOTR(36),ROOTI(36)
      CF=1.0D-04
      H=1000.0
      G=9.8
      SLP=0.1
      S=5.0
      CAPA=.10
    1 READ(5,5)W
    5 FORMAT(F10.5)
      IF(W.GT.14.5)GOTO99
      WRITE(6,10)W
   10 FORMAT(50X,'W=',F10.5,/)
      OMEGA=C.50
   12 OMEGA=OMEGA-.001
      IF(OMEGA.DE.0.0)GOTO1
      A=.5*(1.0/OMEGA+1.0-(CAPA*(OMEGA*OMEGA-1.0))/W)
      AP=.5*(1.0-1.0/OMEGA-(CAPA*(OMEGA*OMEGA-1.0)/(W*S)))
      C(1)=1.0
      C(2)=A
      DO 2 N=3,12
      C(N)=C(N-1)*(A+(N-2))/((N-1)*(N-1))
    2 CONTINUE
      D(1)=A
      D(2)=A*(A+1)/2.0
      DO 3 N=3,12
      D(N)=D(N-1)*(A+(N-1))/((N-1)*N)
    3 CONTINUE
      CP(1)=1.0
      CP(2)=AP/S
      DO 4 N=3,12
      CP(N)=CP(N-1)*(AP+(N-2))/((N-1)*(N-1)*S)
    4 CONTINUE
      DP(1)=AP
      DP(2)=AP*(AP+1)/(2.0*S)
      DO 6 N=3,12
      DP(N)=DP(N-1)*(AP+(N-1))/((N-1)*N*S)
    6 CONTINUE
      DO 7 K=1,12
      CCP(K)=0.0
      DO 7 I=1,12
      DO 7 J=1,12
      IF(I+J.NE.K)GOTO7
```

```
      CCP(K-1)=CCP(K-1)+C(I)*CP(J)
    7 CONTINUE
      DO 8 K=1,12
      CDP(K)=0.0
      DO 8 I=1,12
      DO 8 J=1,12
      IF(I+J.NE.K)GOTO8
      CDP(K-1)=CDP(K-1)+C(I)*DP(J)
    8 CONTINUE
      DO 9 K=1,12
      DCP(K)=0.0
      DO 9 I=1,12
      DO 9 J=1,12
      IF(I+J).NE.K)GOTO9
      DCP(K-1)=DCP(K-1)+D(I)*CP(J)
    9 CONTINUE
      DO 11 N=1,11
      B(N)=CCP(N)-CDP(N)-DCP(N)
   11 CONTINUE
      FCN=B(1)+B(2)*W+B(3)*W*W+B(4)*W**3+B(5)*W**4+B(6)*W**5+B(7)*W**6
     *+B(8)*W**7+B(9)*W**8+B(10)*W**9+B(11)*W**10
      WRITE(6,15)A,AP,OMEGA,W,FCN
   15 FORMAT(2X,4F15.7,D25.16)
      GOTO12
   99 STOP
      END
```

APPENDIX C

DISPERSION RELATION FOR DRONKERS' RECTANGULAR CHANNEL

A. FORMULATION

The equations to be satisfied are:

$$\frac{\partial u}{\partial t} - fv + g\frac{\partial \zeta}{\partial x} = \frac{\partial v}{\partial t} + fu + g\frac{\partial \zeta}{\partial y} = 0 \tag{1}$$

$$h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{\partial \zeta}{\partial t} = 0 \tag{2}$$

Substitution of

$$(u, v, \zeta) = (U, V, \eta)\, e^{-i(my + \sigma t)}$$

into (1) and (2) gives

$$\begin{aligned} i\sigma U - fV + g\frac{d\eta}{dx} &= 0 \\ i\sigma V + fU + img\eta &= 0 \\ h\left(\frac{dU}{dx} + imV\right) + i\sigma\eta &= 0. \end{aligned} \tag{3}$$

Then, for $\sigma^2 \neq f^2$,

$$\begin{aligned} U &= \frac{ig}{\sigma^2 - f^2}\left(mf\eta + \sigma\frac{d\eta}{dx}\right) \\ V &= \frac{-g}{\sigma^2 - f^2}\left(m\sigma\eta + f\frac{d\eta}{dx}\right) \end{aligned} \tag{4}$$

and the ordinary differential equation for η is

$$\frac{d^2\eta}{dx^2} + A^2\eta = 0 \tag{5}$$

where

$$A^2 = \left(\frac{\sigma^2 - f^2}{gh} - m^2\right) \tag{6}$$

with solution of the form

$$\eta = a \sin Ax + b \cos Ax. \tag{7}$$

B. BOUNDARY CONDITIONS

The coordinate system is shown in Figure C1. The flux normal to the channel walls must vanish at the walls.

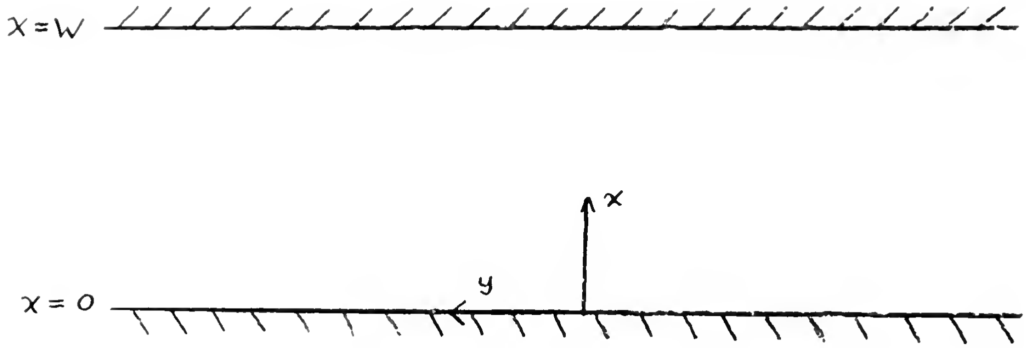

Figure C1

Then (4) gives

$$\frac{mf\eta}{\sigma} + \frac{d\eta}{dx} = 0 \qquad (x = 0, W). \tag{8}$$

Substitution of (7) into (8) yields

$$\left[\frac{m}{AW} + \frac{AW}{m}\right] \tan AW = 0 \tag{9}$$

so that,

$$\tan AW = 0$$

or

$$A^2 = \frac{M^2 \pi^2}{W^2} \qquad (M = 0, 1, 2, \ldots). \tag{10}$$

Then (6) gives the dispersion relation

$$m^2 = \frac{\sigma^2 - f^2}{gh} - \frac{M^2 \pi^2}{W^2}. \tag{11}$$

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Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION 2b. GROUP

3. REPORT TITLE

Long Wave Propagation in a Triangular Channel

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Master's Thesis; October 1969

5. AUTHOR(S) (First name, middle initial, last name)

William Toft Bowman

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
October 1969	47	6

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

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13. ABSTRACT

The purpose of this paper is to examine long wave propagation in a shallow channel of triangular cross section. Solutions for small scale ($f=0$), large scale ($f\neq0$), symmetric and asymmetric channels are obtained. Results are shown to be consistent with earlier work for the fundamental mode ($M=0$), and with edgewave solutions over a gently sloping bottom for the higher modes. A second class of waves (quasi-geostrophic waves) is also obtained when the Coriolis effect is included. The results are compared with those for a rectangular channel.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Triangular Channel						
Edgewaves						
Hypergeometric Equation						
Laguerre Function						
Quasigeostrophic Waves						

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